RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2018
FIRST YEAR (BATCH 2017-20)
MATH FOR ECONOMICS (General)

Date : 24/05/2018
Time : 11.00 am – 2.00 pm
Paper : II
Full Marks : 75

[Use a separate Answer Book for each Group]

Group-A

Answer any seven questions [7×5]

[Notations are their usual meanings]

1. Let $D \subseteq \mathbb{R}$ and two functions $f : D \to \mathbb{R}$ and $g : D \to \mathbb{R}$ is continuous at $c \in D$, then prove that $\left(|f| + g^2\right)$ is continuous at c. 5

2. Show that the function defined as

$$f(x) = \begin{cases} \dfrac{\sin 2x}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$$

has removable discontinuity at $x = 0$. 5

3. A function f is defined on some neighbourhood of c and f is differentiable at c. Prove that

$$\lim_{h \to 0} \frac{f(c+h) - f(c-h)}{2h} = f'(c).$$ 5

4. a) State Rolle's theorem and Lagrange's mean value theorem. 3
   b) Give an example of a differentiable function whose derivative is not continuous. 2

5. Find the series expansion of the function e^x by Maclaurin's theorem. 5

6. Let $f : [a,b] \to \mathbb{R}$ be continuous on $[a,b]$ and $f''(x)$ exists for all $x \in (a,b)$. Prove that there exists a point ξ in (a,b) such that $f(c) = \dfrac{b-c}{b-a} f(a) + \dfrac{c-a}{b-a} f(b) + \dfrac{1}{2}(c-a)(c-b) f''(\xi), \quad a < c < b$. 5

7. A function $f : \mathbb{R} \to \mathbb{R}$ satisfies the condition $|f(x) - f(y)| \leq (x-y)^2$ for all $x, y \in \mathbb{R}$. Prove that f is a constant function on $\mathbb{R}$. 5

8. If c is an interior point of the domain of a function f and $f'(c) = 0$, then prove that the function f has a maximum or minimum value at c according as $f''(c)$ is negative or positive. 5

9. a) What is the geometrical interpretation of Lagrange's mean value theorem. 2
   b) Examine the function $(\sin x + \cos x)$ for extreme values. 3

10. Evaluate: $\lim_{x \to 0} \left(\dfrac{\tan x}{x} \right)^{\frac{1}{x^2}}$. 5

Group-B

Answer any four questions [4×10]

11. a) A linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ maps the basis vectors α, β, γ to $(\alpha+\beta), (\beta+\gamma), \gamma$ respectively. Show that T is an isomorphism. 6
    b) Let V & W be vector spaces over a field $\mathbb{F}$. If a linear mapping $T : V \to W$ be invertible, then prove that the inverse mapping $T^{-1} : W \to V$ is also linear. 4

12. a) Define linear dependence and independence for an infinite set of vectors in a vector space. 5

b) Solve the following system of equations in integers: 5

$$x + 2y + z = 1$$
$$3x + y + 2z = 3$$
$$x + 7y + 2z = 1$$

13. a) Find a basis and the dimension of the subspace W of $\mathbb{R}^3$, where 5

$$W = \{(x, y, z) \in \mathbb{R}^3 | x + y + z = 0\}$$

b) If W_1 and W_2 are two subspaces of a vector space V. Then prove that $W_1 \cap W_2$ is also a subspace of V. 5

14. a) Show that the set S is a subspace of the vector space $C_{\mathbb{R}}[0,1]$ (where $C_{\mathbb{R}}[0,1]$ is the vector space of continuous real-valued functions defined on the closed interval $[0,1]$) if

$$S = \{ f \in C_{\mathbb{R}}[0,1] \mid f(0) = 0, \ f(1) = 0\}$$ 4

b) Let, V & W be vector spaces over a field $\mathbb{F}$. Let, $\{\alpha_1, \cdots, \alpha_n\}$ be a basis of V and $\beta_1, \beta_2, \cdots, \beta_n$ be arbitrarily chosen elements (not necessarily distinct) in W. Then prove that, there exists one and only one linear mapping $T: V \to W$ such that $T(\alpha_i) = \beta_i$ for i = 1, 2, ...,n. 6

15. a) The matrix of a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ with respect to the ordered basis $\{(0,1,1), (1,0,1), (1,1,0)\}$ of $\mathbb{R}^3$ is given by 6

$$\begin{pmatrix} 0 & 3 & 0 \\ 2 & 3 & -2 \\ 2 & -1 & 2 \end{pmatrix}$$. Find T.

b) Prove that the two finite-dimensional vector spaces V & W over a field $\mathbb{F}$ are isomorphic if and only if $\dim V = \dim W$. 4

16. a) Let W_1 and W_2 be two subspaces of a vector space V. Then explain whether $W_1 \cup W_2$ is a subspace of V. 3

b) State the necessary and sufficient condition for a homogeneous system to have non-zero solutions. 2

c) Let V be a vector space of dimension k, where k is an odd positive integer. Is there any linear transformation $T: V \to W$ such that the relation Rank of T = Nulity of T holds? 5

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